ates of the liquidus and solidus isotherms; $z_{liq} = b/R$; $z_{sol} = r_c/R$; $Fo = at/R^2$, $Bi = \alpha R/\lambda$, $K_1 = \mathcal{L}/C(T_{sol} - T_{sol})$, similarity numbers (dimensionless time, cooling criteria, and thermophysical properties of the alloy), $a = \lambda/\rho C$.

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CONFIGURATION OF THERMALLY LOADED COMPONENTS IN ELECTRONIC EQUIPMENT

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Formulation of the Problem. One of the problems of electronic design is that of creating an optimum configuration of components from the thermal viewpoint. By configuration we mean determination of the positions of the modules and components making up a piece of equipment, as well as determination of positions of individual topological elements upon a printed circuit card. Usually component configuration is decided from mounting and connection considerations, but with increased component density and heat liberation ever more attention must be given to thermal criteria. An optimum configuration results in reduction of component temperatures, leading to an increase in reliability of the equipment as a whole. Criteria for evaluating component placements are chosen from increased reliability considerations. Temperature dependences of failure rates have been determined experimentally and can be found in handbooks on reliability [1, 2]. These experimental dependences can be approximated well by exponential functions [3, 4]; however to the accuracy required for practical purposes within a limited temperature range failure intensity can be represented as a linear function of temperature [5]:

$$\lambda(t) = C_1 + C_2 t, \quad (1)$$

where C_1, C_2 are approximation coefficients. Hence the criterion of reducing the net component failure rate leads to a need to reduce the net component temperature:

$$\min \varphi_1 = \min \sum_{j=1}^{n} t_j. \quad (2)$$

In a number of cases it becomes necessary to achieve temperature equalization by reconfiguration of components. This occurs when it is necessary to minimize temperature stresses in a module or decrease electrical imbalance in a circuit caused by differing temperatures of its components etc. [2, 6]. The temperature equalization requirement can be written in the form

$$\min \varphi_2 = \min \sum_{j=1}^{n} (t_j - \bar{t})^2, \ \bar{t} = \frac{1}{n} \sum_{j=1}^{n} t_j. \quad (3)$$

We will now formulate the configuration problem. The electrical connections between the components, their dimensions, and the heat which they liberate are known. A set of limits is specified for temperature, volume, and cost. It is then necessary to create a configuration which will produce an extremum in the chosen criteria for the specified limitations. Two approaches to this problem are possible. In the first the problem is reduced to arranging the components by some algorithm with the goal of minimizing the chosen thermal regime criteria, for example, in the form of Eq. (2) or (3), with limitations on the volume and location of elements. In the second approach the volume of the equipment is minimized with

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limitations on the thermal regime, which can be specified in the form of limits for operating temperatures of the components.

The configuration problem with respect to thermal quality was considered in [5]. With some simplifications the problem was reduced to determination of intercomponent thermal coefficients by solving a linear programming problem with limits on admissible temperature values. As a result an optimal thermal coefficient matrix was obtained, with consideration of which further placement of components was performed. In [7] the problem of arranging the highest number of heat sources within a limited volume for a specified range of admissable component temperatures was considered. The solution method was based on use of a special mathematical apparatus involving hodographs of the dense-configuration vector-function.

Another approach is possible, in which the optimum configuration is found for specified cooling conditions. Formulation of the problem reduces to the following. Let n elements require placement, with each element being characterized by a specified heat liberation P_i ($i = 1, 2, \ldots, n$). Let there also be a fixed number of positions m in each of which any of the components can be mounted. We will assume further that $n = m$. If $m > n$ we may introduce $m - n$ fictitious elements for which the heat liberation is equal to zero. The elements must be arranged in the positions in a manner such that the chosen criterion (for example φ_1 or φ_2) is minimized. Choice of one or the other criterion is determined by the requirements of the specific concrete problem.

Solution of the Configuration Problem. In the general case the temperature of element j can be described in the following form [8]:

$$t_j = \vartheta_j + \vartheta_{jb} + t_m, \tag{4}$$

where t_m is the temperature of the medium surrounding the device: ϑ_j is the overheating of element j caused by the combined action of the arranged heat sources; ϑ_{jb} is the background overheating caused by the action of unarranged (arranged or located according to other quality criteria or designated as such by the designer) heat sources, as well as possible differences in temperatures of the medium surrounding the element under consideration in different directions.

With consideration of Eq. (4) the criterion φ_1 takes on the form

$$\varphi_1 = \sum_{j=1}^{n} \vartheta_j + \sum_{j=1}^{n} \vartheta_{jb} + \sum_{j=1}^{n} t_m. \tag{5}$$

Since the second and third terms are independent of element configuration, criterion (5) can be simplified:

$$\varphi_1' = \sum_{j=1}^{n} \vartheta_j. \tag{6}$$

Commencing from the superposition principle of [8]:

$$\vartheta_j = \sum_{i=1}^{n} \vartheta_{rj} = \sum_{i=1}^{n} P_r F_{ij}, \quad r = r(i), \tag{7}$$

where ϑ_{rj} is the overheating from the heat source located in position i at position j; F_{ij} is the thermal coefficient between points i and j, which is independent of both element power and temperature; r(i) is the configuration function defining the number of the element located in position i.

With consideration of Eq. (7) the configuration criterion takes on the form

$$\varphi_1' = \sum_{j=1}^{n} \sum_{i=1}^{n} P_r F_{ij} = \sum_{i=1}^{n} P_r \sum_{j=1}^{n} F_{ij}. \tag{8}$$

The thermal coefficient matrix can be calculated from the chosen thermal model independent of component arrangement within the established positions. We use the notation

$$S_i = \frac{1}{n} \sum_{j=1}^{n} F_{ij} \tag{9}$$

and write Eq. (8) in the form

$$\varphi_1' = n \sum_{i=1}^{n} P_r S_i. \tag{10}$$

Finally, the configuration problem reduces to seeking a minimum for the following expression:

$$\min \varphi_1' = \min_R \sum_{i=1}^{n} P_r S_i \tag{11}$$

within the set of all possible configurations R. Such a problem is a special case of the problem of linear assignment [9]. A minimum in the criterion φ_1' within the set of all possible configurations R corresponds to arrangement of the vector P in decreasing order, and the vector S in increasing order. From this there follows the following configuration algorithm:

calculate the thermal coefficient matrix F for all installation locations with the chosen thermal model;

calculate the vector S according to Eq. (9);

number the positions in order of increasing characteristic S;

number the elements in order of decreasing heat liberation P;

configure the elements in accordance with this enumeration (first element installed in first position, second in second, etc. for all elements).

The configuration thus obtained provides an exact solution of the problem under consideration. Performing analogous transformations of the criterion φ_2 we obtain

$$\min \varphi_2 = \min_R \left[\sum_{k=1}^{n} \sum_{l=1}^{n} P_m P_g B_{kl} + \sum_{k=1}^{n} P_g V_k\right], \quad m = r(l), \; g = r(k), \tag{12}$$

where

$$B_{kl} = \sum_{j=1}^{n} F_{kj} A_{lj} = \sum_{j=1}^{n} A_{kj} A_{lj}, \tag{13}$$

$$A_{ij} = F_{ij} - S_i, \tag{14}$$

$$V_k = 2 \sum_{j=1}^{n} \vartheta_{j\text{b}} A_{kj}. \tag{15}$$

In the case of uniform temperature background the second term of Eq. (12) vanishes, and we arrive at a special case of the quadratic assignment problem [9]. If nonuniformity of the temperature field is determined mainly by background overheating, then the first term of Eq. (12) can be neglected, and the problem reduces to the linear assignment problem considered above.

In the general case the problem can be solved in two stages. Initially the "inverse configuration" method [9, p. 153] can be used to obtain an initial coarse configuration. To do this each position is assigned a value E_k, calculated with the expression

$$E_k = \sum_{l=1}^{n} B_{kl} + V_k/\overline{P}, \; \overline{P} = \frac{1}{n} \sum_{l=1}^{n} P_l, \; k = 1, 2, \ldots, n. \tag{16}$$

In the second stage we refine the configuration using a paired interchange algorithm. The change in the criterion φ_2 upon exchange of the components located in positions k and ℓ is equal to:

$$\Delta\varphi_2 = \Delta P\,[W_l - W_k + V_l - V_k + \Delta P\,(B_{kk} + B_{ll} - 2B_{kl})], \tag{17}$$

$$g = r(k), \; m = r(l), \; \Delta P = P_g - P_m,$$

$$W_i = 2 \sum_{k=1}^{n} P_g B_{ki}, \quad i = 1, 2, \ldots, n. \tag{18}$$

Finally we have the following configuration algorithm:

calculate the thermal coefficient matrix F for all installation locations with the chosen thermal model;

calculate the matrices A and B with Eqs. (14), (13);

calculate the vectors V and E with Eqs. (15), (16);

number the positions in order of increasing E;

number the elements in order of decreasing power P;

specify the initial configuration in accordance with the element and position enumerations;

calculate the vector W with Eq. (18);

use Eq. (17) to determine the change in the configuration criterion for all possible paired interchanges and select the best interchange. When no interchanges which improve the criterion are possible the configuration is completed, while in the opposite case the interchange is performed and the previous step repeated.

The algorithm described can be used to configure elements on a card, cards within a cardcage, units within a rack, etc.

The algorithms were used to develop a configuration for heat-liberating elements on a printed circuit card. The thermal model of the card with local heat sources was presented in [10]. From 30 to 60 components were mounted. Calculation time required on an ES-1022 computer was about 2 min. Use of the algorithm has revealed that in a number of cases the initial configuration variant was far from optimal. The question arises of the need for calculating an initial configuration, since the latter sometimes proves to be no better than a randomly determined configuration. However, even in this case the initial configuration calculation should not be eliminated, since the increase in the total volume of calculations required is insignificant.

NOTATION

P_j, t_j, heat liberation and temperature of element j; φ_1, φ_2, criteria evaluating quality of configuration; t_m, temperature of medium; F_{ij}, thermal coefficient between positions i and j; r(i), configuration function defining the number of the component located in position i; n, number of elements to be configured; R, set of all possible permutations.

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